Preface

This book is meant for a one semester course on *Functional Analysis* for about 40 contact hours of 50 minutes each for MSc level students.

Functional analysis is the study of linear spaces and maps between these spaces when the spaces are endowed with additional structures on them, such as *norms*, *metrics* or *topologies*. In this course, we essentially study linear spaces with norms, and linear operators between linear spaces.

We assume that the reader is familiar with basic concepts from Linear Algebra and Real Analysis at the levels dealt in references Halmos [4] and Rudin [9], respectively, and we shall freely use them without much references to them. However, occasionally, for the sake of completeness of presentation, we shall state the required results explicitly. Problems are given at the end of each chapter; most of the problems are meant to supplement the results given in that chapter.

The treatment of the topics in this book is only of introductory nature, which is only meant to make the student to look into other standard books on Functional Analysis for gathering more information and knowledge in the subject. An often referred book for some of the stated results without proof is the author's own book [5].

This book contains four distinct chapters. Chapter 1 deals with normed linear spaces and their properties, including many properties of *Banach and Hilbert spaces*. Chapter 2 is on linear operators between normed linear spaces. Chapter 3 is christened as *Important Theorems* as it deals with some of the most important theorems in Functional Analysis such as *Closed Graph Theorem, Open Mapping Theorem, Uniform Boundedness Principle, Hahn-Banach Theorem* and *Riesz Representation Theorem*. A bit advanced topic, namely, *Spectral Results* is dealt in the final chapter. In this chapter, the spectral theorem is dealt only for compact self adjoint operators; spectral theorem for self adjoint operators is omitted very reluctantly, due to constrains of being an introductory course.

Notations:

- We shall use standard notations of set theory as well as standard symbols

$$\mathbb{N}, \quad \mathbb{Z}, \quad \mathbb{Q}, \quad \mathbb{R}, \quad \mathbb{C}$$

  for the set of *positive integers*, the set of *integers*, the set of *rational numbers*, the set of *real numbers*, and the set of *complex numbers*, respectively.

- We shall use the logical symbols

$$\Longrightarrow, \qquad \Longleftrightarrow$$

  for *only if* and *if and only if*, respectively.

- We shall also write

  (a) $x, y \in S$ in place of $\{x, y\} \subseteq S$,

  (b) $i = 1, \ldots, n$ in place of $i \in \{1, \ldots, n\}$.

- Throughout the course, we shall denote by $\mathbb{K}$ the set $\mathbb{R}$ of all real numbers or the set $\mathbb{C}$ of all complex numbers. Linear spaces that we consider are over the field $\mathbb{K}$.

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I would be very grateful for critical comments, corrections, and suggestions from the users of this book.

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