References

[1] C.D. Aliprantis and O. Burkinshwa, *Priciples of Real Analysis*, Academic Press (Elsvier), 1998.

[2] J. Dixmier, Sur les bases orthonormales dans les espaces prehilbertiens, *Acta Sci. Math. Szeged*, **15** (1953) 29-30.

[3] S.R. Ghorpade and B.V. Limaye, *A Course in Calculus and Real Analysis*, Springer, 2006.

[4] P.R. Halmos, *Finite Dimensional Vector Spaces*, Springer, 1987.

[5] M.T. Nair, *Funtional Analyis: A First Course*, Prentice Hall of India, 2002 (Third Print, PHI Learning, New Delhi, 2010).

[6] M.T. Nair, *Linear Operator Equations: Approximation and regularization*, World Scientific, 2009.

[7] S. Ramaswamy and CT. Ramaswamy, Deduction of one from another of three major theorems in functional analysis, (Preprint), 2011.

[8] H.L. Royden, *Real Analysis*, 3rd ed., Prentice-Hall of India, New Delhi, 1995.

[9] W. Rudin, *Principles of Mathematical Analysis*, 3rd ed., McGraw-Hill, New York, 1976.

[10] Rudin, W., *Real and Complex Analysis*, 3rd ed., McGraw-Hill International Editions, New York, 1987.

Index

$B(\Omega)$, 4
$C_0(\mathbb{R})$, 62
$C_b(\Omega)$, 4, 35
$C_c(\mathbb{R})$, 62
$L^\infty[a,b]$, 94
$L^p[a,b]$, 46, 93
ℓ^∞, 5
$\ell^\infty(\Omega)$, 21
ℓ^p, 22
c, 5
c_0, 5
c_{00}, 5
$\mathcal{P}_n$, 3
$\mathcal{P}_n(J)$, 4

absolutely convergent, 38
adjoint, 85, 87
Arzela-Ascoli theorem, 99

Banach space, 15, 27
basis, 6
    ordered, 7
bounded below, 77
bounded linear operator, 66

canonical isometry, 128
Cauchy sequence, 14
closed operator, 106
compact operator, 97
complex-linear functional, 123
compression spectrum, 142
conjugate exponents, 22
conjugate linear, 80
convergent series, 38
converges, 14

dense, 48
dimension, 7
dual, 68
dual space, 68

equi-continuous, 99
equivalent norms, 28
essentially bounded, 46
Euclidean
    norm, 18
Euclidean norm, 13

finite dimensional, 7
Fourier coefficients, 55

Hölders inequality, 46
Hilbert space, 26

inner product, 25
inner product space, 25
invariant, 154
invariant subspace, 154
isometry
    canonical isometry, 128
isomorphic, 9

linear
    combination, 6
    functional, 9
    isomorphism, 9
    operator, 9, 65
    space, 1
    transformation, 9

linearly
dependent, 6
independent, 6
isomorphic, 9

matrix representation, 11
Minkowski inequality, 46

norm, 13
normal, 147
operator, 87
numerical
radius, 150
range, 150

orthogonal, 47
compliment, 47
projection, 58, 74, 83
set, 48
orthonormal
basis, 49
sequence, 48
set, 48

partial sum, 38
pointwise bounded, 99
positive
operator, 150
self adjoint, 150
product norm, 107
projection
operator, 58
theorem, 56

quotient space, 41

real-linear functional, 123
reflexive space, 129
relatively compact, 96
resolvent set, 141
Riesz Representation Theorem, 79

scalars, 2
Schwarz inequality, 25
self adjoint, 147
operator, 83
self-adjoint, 147
semi-norm, 40
separable, 48
series, 38
sesquilinear form, 81
singular
system, 160
value decomposition, 160
value representation, 158
values, 158, 160
vectors, 160
spectral radius, 145
spectrum, 141
standard basis, 8
stronger norm, 29
subspace, 2

unitary, 147
unitary operator, 87

vector space, 2
vectors, 2

weaker norm, 29